Methods for Analysis of Spatio-Temporal Bluetooth Tracking Data

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ABSTRACT *Analysis of people's movements represented by continuous sequences of spatio-temporal data tuples have received lots of attention in recent years. The focus of those studies was mostly GPS data recorded on a constant sample rate. However, the creation of intelligent location-aware models and environments also requires reliable localization in indoor environments as well as in mixed indoor/outdoor scenarios. In these cases, signal loss makes usage of GPS infeasible; therefore other recording technologies evolved. Our approach is analysis of episodic movement data. This data contains some uncertainties among time (continuity), space (accuracy), and the number of recorded objects (coverage). Prominent examples of episodic movement data are spatio-temporal activity logs, cell-based tracking data, and billing records. To give one detailed example, Bluetooth tracking monitors the presence of mobile phones and intercoms within a sensor's footprints. Usage of multiple sensors provides flows among the sensors. Most existing data mining algorithms use interpolation and therefore are infeasible for this kind of data. For example, speed and movement direction cannot be derived directly from episodic data; trajectories may not be depicted as a continuous line; and densities cannot be computed. Still, the data hold much information on group movement. Our approach is to aggregate movement in order to overcome the uncertainties. Deriving a number of objects for the spatio-temporal compartments and transitions among them gives interesting insights on the spatio-temporal behavior of moving objects. As a next step to support analysts, we propose clustering the spatio-temporal presence and flow situations. This work focuses as well on creation of a descriptive probability model for the movement based on Spatial Bayesian Networks. We present our methods on a real world data set collected during a football game in Nîmes, France in June 2011.*

KEYWORDS *Bluetooth Tracking; Mobility Mining; Event Monitoring*

Introduction

Major airports, arenas, and stadiums are designed to attract thousands or billions of visitors each year. One trend has been the building of larger infrastructures (airports, stadiums) while another trend is the growing number of visitors at major events. This hazardous development has led to devastating disasters (for example, the Loveparade stampede in Duisburg, Germany in 2010). Thus, visitor monitoring in complex facilities became an important subject. But

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understanding the movement behavior, identification of attractors and distractors, and determination of waiting times, as well as localization of congestions and bottle-necks also gives insights into visitor preferences and motivations at a particular site or event. Knowing such detailed information about indoor pedestrian behavior also gives a location-based performance indicator for different locations inside buildings. Various locations and attractions can be ranked by their popularity, safety, or frequency. Recently evolved Bluetooth tracking (Bruno and Delmastro, 2003) became the state-of-the-art method for the combined indoor/outdoor monitoring of pedestrian movement (Andrienko et al., 2012; Hagemann and Weinzerl, 2008; Leitinger et al., 2010; Liebig and Kemloh Wagoum, 2012; Stange et al., 2011; Versichele, 2012a, 2012b).

Visual exploration of the collected partial trajectories gives indispensable insights to an event (Liebig and Kemloh Wagoum, 2012; Utsch and Liebig, 2012). For the determination of visitor preferences or identification of potential hazards, it is also necessary to discover the dependencies, correlations, and patterns among the movements. Therefore, this work tackles the computationally enabled visual exploration of a massive Bluetooth tracking dataset for inner dependencies which was the result of the non-random movement of people. Existing approaches, e.g. direct database access or use of a trajectory data warehouse (TDW) (Orlando et al. 2007; Raffaetà et al., 2011), are unfeasible as the first one requires powerful database hosts and the second pre-aggregates the data and prevents further analysis. Our proposed method contains two stages. We represent the massive movement data by an easy-to-handle descriptive model, namely a Spatial Bayesian Network (SBN) (Liebig et al., 2008, 2009). This probabilistic model denotes the conditional probabilities among visits to discrete locations and thus holds all required information in a compact format for further querying. In step 2 we use the previously trained SBN for visual analysis and depict the probability distributions on three-dimensional thematic maps.

Analysis of people's movements represented by continuous sequences of spatio-temporal data tuples has received considerable attention in recent years (Giannotti and Pedreschi, 2008). The focus of the studies was mostly GPS data recorded on a constant sample rate. However, creation of intelligent, location-aware models and environments also requires reliable localization in indoor environments as well as in mixed indoor/outdoor scenarios. In these cases, signal loss makes the use of GPS infeasible, therefore other recording technologies evolved.

Besides video surveillance and 3D laser scan technologies, Bluetooth tracking technology emerged as a passive pedestrian tracking technology. Bluetooth tracking provides three major benefits: (1) no additional scaffoldings are required, (2) no adjustment of the sensor beacons is necessary, and (3) the sensors are seamlessly usable indoors and outdoors. These circumstances have made Bluetooth tracking the preferred application for passive pedestrian tracking. After a detailed literature review, this work addresses the analysis of a soccer stadium data set collected in Nîmes, France in June 2011 (Liebig and Kemloh Wagoum, 2012).

The remainder of the paper proceeds as follows. The upcoming section gives an overview of related Bluetooth tracking work and introduces pedestrian mobility analysis with Bluetooth tracking data. After that, we give a brief summary of Spatial Bayesian Networks and present our approach. Finally, we conclude and discuss an outlook on future research.

Bluetooth Scanner Technology

The Bluetooth sensors (also scanners, transceivers, or beacons) throughout this work are inspired by Bruno and Delmastro (2003) and are assembled using a microcomputer and three USB Bluetooth antennas. A Linux-based software activates the antenna's inquiry mode and logs the hashed MAC addresses of the detected devices.[1] Hashing the MAC address prevents re-identification of devices in other datasets and increases privacy for the tracked persons. A common hash function is the sha256 algorithm (NIST, 2002) as its reverse function is very hard to compute. Thus, the scan interval of approximately 10.24s (Bruno and Delmastro, 2003) is (theoretically) reduced to its third. This time span is required for frequency hopping and device discovery (Bluetooth SIG, 2004). However, in practice the antennas are not synchronous and data points are not recorded with a constant frequency. The recorded data log entries consists of:

[time stamp], [sensor ID], [sha256(MAC)], [signal strength].

The two different antenna types which are used have a range of either 20 m or 100 m. Nevertheless, since the inquiry mode requires communication in both directions (from the sensor to the mobile device and vice versa) (Bluetooth SIG, 2004) the size of the sensor footprint not only depends on the sensors antennas but also on the antenna in the mobile device (and its configuration). Thus, lower sensor ranges of about 20 m are assumed. The recorded unique MAC addresses of the Bluetooth antennas of the mobile devices consist of six bytes. Three of them depend on the vendor and provide valuable information for analysis as the type of the Bluetooth device can roughly be estimated. The ratio of detected people varies over space and time due to different Bluetooth visibility rates.

Data Analysis Workflow

A first systematic workflow for mobility mining from Bluetooth tracking data for a pedestrian monitoring scenario was introduced in Stange et al. (2011). However, this primary framework was specialized for an event monitoring scenario. Recently, in Liebig et al. (2012), the workflow was extended to a more general one, reflecting the phases of the knowledge discovery process (Fayyad et al. 1996). In their work, Bluetooth-based mobility studies are conducted in three consecutive phases consisting of five steps.

- The *field study phase* is performed during (1) survey design and (2) data collection.
- The second *visual analysis phase* is conducted within the (3) data preparation, aggregation and (4) visual analysis.
- In the *knowledge discovery phase* we conduct the (5) pedestrian analysis step.

Survey Design and Data Collection. The field study phase comprises the survey design and data collection step. For the *survey* design, the number and location of the Bluetooth scanners needs to be derived from the application requirements. Furthermore, in the case of non-stationary but moving Bluetooth scanners, their spatio-temporal distribution needs to be configured (Naini et al., 2011).

The *data collection* could be done offline or online. Offline data collection implies batch processing of the recorded data, whereas online communication allows for real-time analysis.

Data Pre-Processing and Filtering. Succeeding the data collection, recorded data log entries need to be filtered to omit noisy or unwanted entries. For example, in the case of pedestrian tracking, it is useful to get rid of vehicular entries caused by car multimedia equipment. The filtering could be performed on all dimensions of the recorded tuples: space, signal strength (for higher granularity of space), time, and vendor (according to MAC address). The filtering is described in detail in Andrienko et al. (2012) and Stange et al. (2011).

Data Aggregation and Visual Analysis. As described in the literature, after the data have been reduced to the relevant entries, the next step is triggered by the monitoring task. As Bluetooth tracking data is episodic movement data (as it contains the common uncertainties on continuity, accuracy, and coverage) spatio-temporal aggregation is a common step, described in the literature (Andrienko et al., 2012; Liebig et al., 2012). However, aggregation time intervals need to be chosen according to the monitoring task, and the literature varies from three-minute aggregation (Utsch and Liebig, 2012) for fast microscopic localization tasks to aggregation of complete days (Liebig et al., 2012) for the estimation of average daily traffic (ADT). The aggregation could be performed on arbitrary events denoted from the movement data. The literature describes *visit* events $<o,p,t>$ occurring whenever a Bluetooth device o is in a particular location p for a time period exceeding t. Another common event type are *moves* between two locations: $<o,pi,pj>$ which occur if both places pi and pj are visited consecutively (Liebig et al., 2013).

The aggregation of these events in space and time returns object counts for flow (flow counts) and visits (presence counts) which correspond to the macroscopic values of movement (Andrienko et al., 2012). Visualizations of the aggregated values help for data understanding and for coarsening filters of the preprocessing step (Andrienko et al., 2012).

Mobility Data Analysis

Similar to monitoring technologies, the models for pedestrian monitoring distinguish between *microscopic* and *macroscopic* aspects of mobility (Hägerstrand, 1974). Whereas microscopic models describe individual behavior and provide trajectories for them, macroscopic models aim at modeling the moving population and use values such as density, quantity, or speed to characterize pedestrian flows. Both of these views on movement are closely related as macroscopic values can be derived by aggregation from microscopic ones (Hägerstrand, 1974).

Microscopic Mobility Analysis

The analysis of microscopic pedestrian movement can make use of the recorded radio signal strengths (Utsch and Liebig, 2012) in order to achieve an accurate position and movement representation of the enabled Bluetooth devices. In their work, which makes use of fingerprinting with a k-Nearest Neighbor algorithm, the achieved positioning precision is about four meters.

Another approach, presented in Liebig and Kemloh Wagoum (2012), makes use of proximity sensing and hands the generated episodic position readings to an agent-based simulation of pedestrian mobility (in this case the Generalized Centrifugal Force Model (Chraibi et al., 2010)) in order to achieve microscopic values on pedestrian mobility. Triggered by the episodical readings of people's presence, the simulation generates a reasonable assumption on the pedestrian distribution in reality.

Macroscopic Mobility Analysis

The proximity-based tracking of Bluetooth devices is applied mostly as valuable temporal information about stay times and transition times. Spatial information on movement sequences and quantities is generated directly.

The analysis of zoo visitors presented in recent work (Ellersiek et al., 2012) distinguishes between position-based and path-based analyses. However, since scanners may not be placed at any arbitrary position, the estimation of the macroscopic values for unobserved locations is necessary. Recent work that uses the same zoo dataset makes use of sequence movement patterns in order to estimate accurate pedestrian quantities (Liebig et al., 2012). Their method is based on Gaussian Process regression methods, including a random walk-based kernel function that represents the movement patterns within a site.

In contrast to these stationary studies, moving sensors are used to track visitor presence along a race track (Versichele et al., 2012a, 2012b, 2012c). In Naini et al. (2011) moving sensors are used to estimate the total number of visitors in a bounded area.

Exemplary Analysis

Previous sections highlighted recent development in Bluetooth tracking and its analysis. In this paper we perform tests on a real-world Bluetooth tracking dataset collected during a soccer match at the Stade des Costières, Nîmes (France) (Liebig and Kemloh Wagoum, 2012). The data were collected using 15 Bluetooth beacons (Bruno and Delmastro, 2003) at various locations in the stadium (See Figure 1). The numbers in the picture denote our sensor IDs and are thus not sequential.

During this study, 14 percent of the visitors (553 of 3,898 persons) were recorded. We conducted a detailed study of stadium visitors during a match in a German multi-purpose arena (Liebig, 2013). There, we compared 15-minute aggregates to the data of the electronic entry control. At all gates of the stadium, the recorded data had a high correlation of about 0.97. Thus, the recorded data were representative.

The recorded Bluetooth tracking dataset contains sequence movement patterns. The most frequent movement pattern with more than one location starts at the main entrance (in the upper left corner in Figure 1) and ends at a tribune (locations at the sides of the stadium, compare also, Figure 2). The movement in the stadium thus is not a random walk but aims at a target. These individual movement preferences cause correlations among the sensor readings. In Liebig et al. (2013) we utilized these movement patterns for sensor placement and data imputation at unobserved locations.

Next, we visually explored the correlations contained in the soccer dataset (Liebig and Kemloh Wagoum, 2012). The visual analysis of movement dependencies among discrete regions is the subject of our previous work (presented in Liebig et al., 2008, 2009). There, the contained dependencies are represented by a Spatial Bayesian Network that connects the different regions by directed

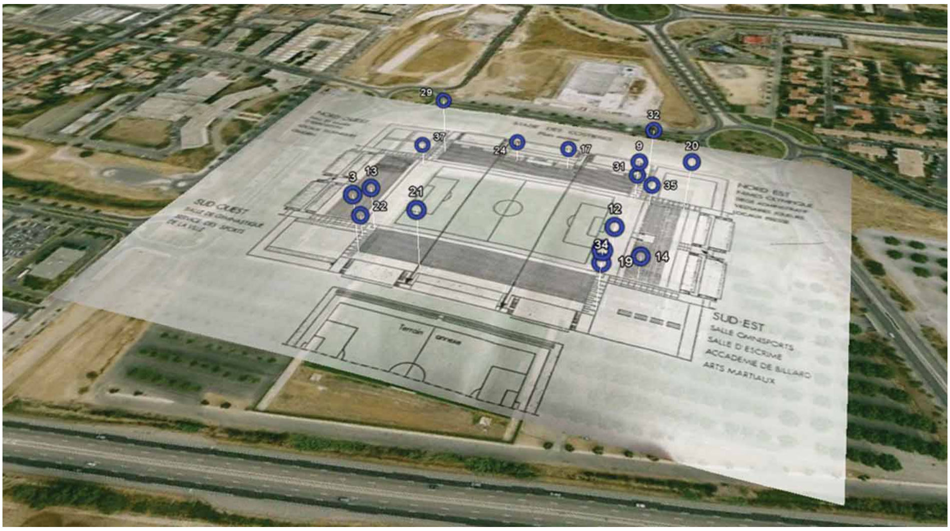

Figure 1: 3D Sensor Placement at Stade des Costières, Nîmes (France) August 5, 2011

edges and associated conditional probability tables. In the results, queries for co-visits of spatial regions given arbitrary (positive or negative) evidences can be answered. Next, we apply this method (Liebig et al., 2008) to the presented dataset and study the contained movement preferences in detail (See Figure 2).

For visualization of the three-dimensional dependencies, we create a Voronoi Dirichlet tessellation (Dirichlet, 1850; Voronoi, 1908) of a three-dimensional stadium model.

Materials to the resulting geometries (color and opacity) are assigned according to the probability distribution computed by the Spatial Bayesian Network. Figure 2 depicts the results of the Spatial Bayesian Network for four different queries. Red colors indicate a high visit probability; blue colors indicate a low probability. The yellow arrows in the picture mark the points of positive evidence. The picture A (in the upper-left corner) depicts the probability distribution given the evidence that the sensor at the ground floor (sensor 34 for

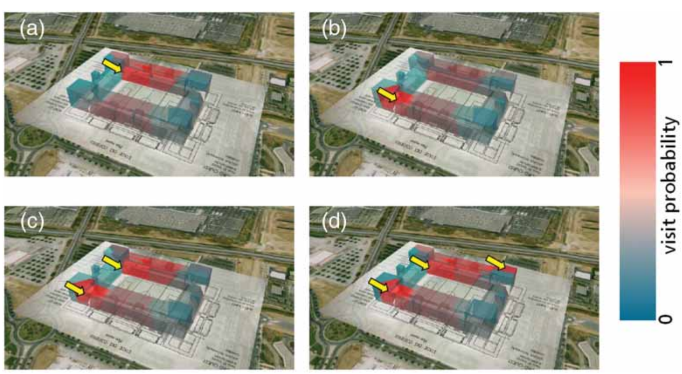

Figure 2: Visual representation of the spatial correlations in the soccer dataset: yellow arrow denotes the evidence of the query.

comparison with Figure 1) has been visited. It is remarkable that the probability on this side of the stadium is high but low in most of the other parts. In conjunction with the VIP rooms (lower left corner of Figure 1) some tribunes possess a relatively high probability (depicted at the bottom of Figure 2). These places are visited by the catering staff and prominent visitors from all tribunes after the match.

In the next step we examine the impact of the staff and prominent guests by changing the evidence to a restricted entry within the Spatial Bayesian Network. Results are depicted in Figure 2b. All paths that have been used by the catering crew and safety deputies are inked in red which denotes a high probability of movement. The shops possess a relatively high probability. They were located in the uppermost floor of the two towers in the left side of the picture and also in the VIP lounges. Safety deputies helped us during data collection, thus it can be seen to the right that they visited sensor location three (top of the upper left tower, compare Figure 2) in order to check its presence. In the bottom of Figure 2 we combine multiple points of evidence within the query. To the left (See Figure 2[c]) is a visualization of the combined probability of the visitors at the entry to the major tribune and to the VIP entry. The visitors selected by this query disperse through the major tribunes and within the VIP rooms. By further addition of evidence at sensor location three, the places considered so far reach their highest conditional probability. Most likely, this untypical movement pattern depicted in Figure 2d was our movement for maintenance of the sensors. Though the movement frequency for maintenance is low with respect to the overall movement, it is filtered by this query and pops up under the posed conditions. The tribune to the left shows a very low probability as it could not be traversed. The tribune on the right was open for traversing before the match began. Thus, our visual analysis reflects these circumstances and helps to understand movement behavior contained in the dataset.

After visual analysis of the recorded spatial movement correlations our further visual analysis focuses on the temporal analysis of the dataset. Since episodic movement data contain uncertainties on individual movement, the proposed approach in Andrienko et al. (2012) is the spatio-temporal aggregation of *presence* and *moves*. These results in presence and flow situations which denote for a time interval dt the total number of *visits* for each discrete location as well as the total number of *moves* among pairs of locations. Thus, the soccer dataset (Liebig and Kemloh Wagoum, 2012) is automatically divided into three consecutive time intervals (arrival, match, departure) derived from the clustering of presence and flows (See Figure 3). In this picture the lines represent the number of persons per scanner (See Figure 3a) or the numbers of persons per link among two locations (See Figure 3b). The background coloring of the Figure utilizes Sammon's mapping (Sammon, 1969). Sammon's projection maps high dimensional points on a two-dimensional color-plane; in the results, similar points are assigned similar colors and vice versa. We apply this method to the flow and presence vectors at a particular time and achieve a color for each of these vectors which is plotted in the background of Figure 3. Based on the achieved visual analysis of the flow data (depicted in Figure 3) characteristic time-spans are (14:00, 20:00, 21:45, 22:00). These time intervals correspond to the three different consecutive phases of the match: arrival of the visitors, match, and the departure after the match. Note that in Figure 3b (which analyzes the moves of the visitors) even the break of the match is visible. Movement of the stadium visitors differs in

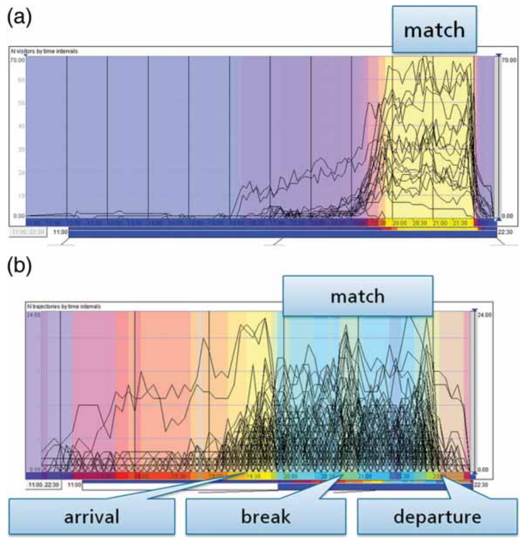

Figure 3: Clustering of (a) presence and (b) flow situations over time

each of these time spans from its successive time interval (indicated by different colors in Figure 3b).

Conclusion and Summary

This work gave an overview of Bluetooth tracking and related literature. We tackled the task to explore and analyze spatio-temporal Bluetooth tracking data. The question is of high interest as Bluetooth tracking is nowadays used for various pedestrian monitoring applications. The challenge related to this task is the three-dimensionality of the movement data. Thus, we created a three-dimensional Dirichlet-Voronoi tessellation of the building based on the positions of the sensors. The visualization was integrated using OGC compliant interfaces and a web service. This allows easy integration into other software modules.

Another challenge, the dependency analysis of the recorded movement data, was addressed using Spatial Bayesian Networks as an intermediate data structure which holds just the required data instead of complete trajectories. Once the model is built, querying is fast and flexible and overcomes the drawbacks of existing methods that rely on random memory access or aggregation (TDW). The proposed methods were integrated and tested using an event monitoring case. Recorded data was analyzed in order to identify and reconstruct pedestrian movement. Analysis of the inner-trajectory correlations revealed in-traversable tribunes as well as visitor preferences.

The analysis of the recorded data is useful to the stadium for understanding people's behavior and preferences. Thus, the phases of the event can be analyzed. As an example, the flow of the people during the break is of interest to increase the sales volume of the snack bars. For safety purposes, the normal user behavior could be used as expert knowledge on the preferred movement behavior. The clustering of presence and flow situations is important for testing the applicability of movement models which are trained for particular conditions.

Our study shows that Bluetooth tracking is a valid approach for recording episodic movement data about people's movements. Nevertheless, many influences make this uncertain (e.g., number of people with enabled Bluetooth devices, locations of the sensors). Therefore, in every application, the representativeness (spatial and temporal) has to be checked in a pre-study by comparison to an alternative (tracking or counting) data source.

Future Work

Episodic movement data are quite frequent, and more methods for its analysis are needed. During the field study phase, more automatic methods are required that would help in making decisions about the placement of sensors. We describe an approach for reduction of required sensors in Liebig (2013). Furthermore a detailed analysis of the Bluetooth representativeness will be conducted; first results are in Liebig (2013).

Note

1. A similar software for Bluetooth Tracking is published by the University of Ghent at https://github.com/Rulus/Gyrid with GPL license, last accessed June 30, 2012.

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Bibliography

N. Andrienko, G. Andrienko, H. Stange, T. Liebig and D. Hecker, “Visual Analytics for Understanding Spatial Situations from Episodic Movement Data,” *KI - Künstliche Intelligenz* 26:3 (2012) 241–251.

Bluetooth SIG. *Specification of the Bluetooth System* (November, 2004) <http://www.bluetooth.com> Accessed June 21, 2012.

R. Bruno and F. Delmastro, “Design and Analysis of a Bluetooth-Based Indoor Localization System,” In Personal Wireless Communications, IFIPTC6 8th International Conference, PWC 2003 (Venice, Italy, 2003).

M. Chraibi, A. Seyfried and A. Schadschneider, “The Generalized Centrifugal Force Model for Pedestrian Dynamics,” *Physical Review E* 82 (2010) 046111. doi:10.1103/PhysRevE.82.046111.

G.L. Dirichlet, “Über die Reduktion der positiven quadratischen Formen mit drei unbestimmten ganzen Zahlen,” *Journal für die Reine und Angewandte Mathematik* 40 (1850) 209–227.

T. Ellersiek, T. Liebig, D. Hecker and C. Körner, “Analyze von raumzeitlichen Bewegungsmustern auf Basis von Bluetooth-Sensoren,” Angewandte Geoinformatik 2012 - Beitr•age zum 24. AGIT-Symposium Salzburg (Wichmann, Berlin, 2012, pp. 260–269).

U. Fayyad, G. Piatetsky-Shapiro and P. Smyth, “From Data Mining to Knowledge Discovery in Databases,” *AI Magazine* 17 (1996) 37–54.

F. Giannotti and D. Pedreschi, *Mobility, Data Mining and Privacy - Geographic Knowledge Discovery* (Heidelberg: Springer, 2008).

W. Hagemann and J. Weinzerl, Automatische Erfassung von Umsteigern per Bluetooth-Technologie. In: Nahverkerspraxis. Springer Berlin Heidelberg, 2008.

T. Hägerstrand, “Tiidsgeografisk Beskrivning. Syfte och Postulat,” *Svensk Geografisk Arsbok* 50 (1974) 86–94.

S. Leitinger, S. Gröchenig, S. Pavelka and M. Wimmer, “Erfassung von personenströmen mit der bluetooth-tracking-technologie,” Angewandte Geoinformatik 2010 - Beiträge zum 22. AGIT-Symposium Salzburg, 15th Edition (Wichmann, Berlin, 2010, pp. 220–225).

T. Liebig, Relating mobility patterns to socio-demographic profiles, PhD Thesis, ULB Bonn, 2013.

T. Liebig and A.U. Kemloh Wagoum, “Modeling Microscopic Pedestrian Mobility using Bluetooth,” Proc. of the Fourth International Conference on Agents and Artificial Intelligence - ICAART’12 (SciTePress, 2012, pp. 270–275).

T. Liebig, C. Körner and M. May, “Scalable Sparse Bayesian Network Learning for Spatial Applications,” ICDM Workshops, IEEE Computer Society (2008, pp. 420–425).

T. Liebig, C. Körner and M. May, “Fast Visual Trajectory Analysis using Spatial Bayesian Networks,” ICDM Workshops, IEEE Computer Society (2009, pp. 668–673).

T. Liebig, Z. Xu and M. May, “Incorporating Mobility Patterns in Pedestrian Quantity Estimation and Sensor Placement,” Proceedings of the First International Workshop on Citizen Sensor Networks, LNAI 7685 (Springer, 2013, pp. 67–80).

T. Liebig, Z. Xu, M. May and S. Wrobel, “Pedestrian Quantity Estimation with Trajectory Patterns,” Proceedings of the European Conference on Machine Learning and Principles and Practice of Knowledge Discovery in Databases ECML PKDD 2012, Part II, LNCS 7524 (Springer, 2012, pp. 629–643).

F.M. Naini, O. Dousse, P. Thiran and M. Vetterli, “Population Size Estimation using a Few Individuals as Agents,” Proceedings of the International Symposium on Information Theory, IEEE (2011, pp. 2499–2503).

NIST, National Institute of Standards and Technology, Secure Hash Standard, National Institute of Standards and Technology, Washington, Federal Information Processing Standard 180-2 (2002).

S. Orlando, R. Orsini, A. Raffaetà, A. Roncato and C. Silvestri, “Trajectory Data Warehouses: Design and Implementation Issues,” *Journal of Computing Science and Engineering* 1: (2) (2007) 240–261.

A. Raffaetà, L. Leonardi, G. Marketos, G.L. Andrienko, N.V. Andrienko, E. Frentzos, N. Giatrakos, S. Orlando, N. Pelekis, A. Roncato and C. Silvestri, “Visual Mobility Analysis Using t-warehouse,” *International Journal of Data Warehousing and Mining* 7: (1) (2011) 1–23.

J.W. Sammon, “A Nonlinear Mapping for Data Structure Analysis,” *IEEE Transaction on Computers* 18: (5) (1969) 401–409.

H. Stange, T. Liebig, D. Hecker, G. Andrienko and N. Andrienko, Analytical workflow of monitoring human mobility in big event settings using Bluetooth, in: ISA 2011, ACM, (2011), pp. 51–58.

P. Utsch and T. Liebig, “Monitoring Microscopic Pedestrian Mobility Using Bluetooth,” Proceedings of the 8th International Conference on Intelligent Environments (IEEE Press, 2012, pp. 173–177).

M. Versichele, R. Huybrechts, T. Neutens and N. Van de Weghe, “Intelligent Event Management with Bluetooth Sensor Networks,” Proceedings of the 8th International Conference on Intelligent Environments (IEEE Press, 2012a, pp. 311–314).

M. Versichele, N. Neutens, S. Goudeseune, F. Van Bossche and N. Van de Weghe, Mobile Mapping of Sporting Event Spectators Using Bluetooth Sensors: Tour of Flanders (2011), Sensors, (2012b)10, 14196–14213.

M. Versichele, T. Neutens, M. Delafontaine and N. Van de Weghe, "The Use of Bluetooth for Analyzing Spatiotemporal Dynamics of Human Movement at Mass Events: A Case Study of the Ghent Festivities," *Applied Geography* 32: (2) (2012c) 208–220.

G. Voronoi, "Nouvelles applications des paramètres continus à la théorie des formes quadratiques. Deuxième mémoire. Recherches sur les parallélloèdres primitifs," *Journal für die reine und angewandte Mathematik (Crelle's Journal)* 134 (1908) 198–287.